Issues and affordances in studying children's drawings with a mathematical eye

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In this third consecutive MERGA symposium focused on young children's drawings, three separate groups of researchers discuss the benefits and issues of using drawings as a source of data in their studies. Although drawings are ubiquitous in early years classrooms and in studies of children's learning, there is no comprehensive framework for analysing children's drawings in mathematical contexts. The overarching purpose of these symposiums has been to explore the qualitative methods that researchers have developed in their distinct projects and advance our critical perspectives on interpreting drawings and understanding the role they can play in children's learning of mathematics.

Broadly, the researchers view drawings as an external representation of mathematical concepts, mathematical thinking, or perceptions of mathematical contexts. Typically, researchers trust that children's drawings express to some extent the developing internal systems of the child, including the affective domain. In studying the interplay between children's internal and external representations, researchers must grapple with the ambiguities of interpreting representational drawing, as explained in quotation below.

> "Internal systems, … include students' personal symbolization constructs and assignments of meaning to mathematical notations, as well as their natural language, their visual imagery and spatial representation, their problem-solving strategies and heuristics, and (very important) their affect in relation to mathematics. The *interaction* between internal and external representation is fundamental to effective teaching and learning. Whatever meanings and interpretations the teacher may bring to an external representation, it is the nature of the student's developing internal representation that must remain of primary interest." (Goldin & Shteingold, 2001, p.2).

In this symposium, as well as sharing results from recent research, the authors reflect on some of the issues and affordances in studying children's drawings with a mathematical eye.

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Chair & Discussant: Jennifer Way

Paper 1: Jill Cheeseman, Ann Downton, Anne Roche & Sarah Ferguson *Drawings reveal young students' multiplicative visualisation*

Paper 2: Katherin Cartwright, Janette Bobis & Jennifer Way *Investigating students' drawings as communication and representation modes of mathematical fluency.*

Paper 3: Kate Quane, Mohan Chinnappan & Sven Trenholm *Children's drawings as a source of data to examine attitudes towards mathematics: Methodological affordances and issues*

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Drawings reveal young students' multiplicative visualisation

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In the context of a multiplicative problem, our study investigated young children's ability to visualise and draw equal groups. This paper reports the results obtained from 18 Australian children in their first year of school (age 5-6 years). The task *12 Little Ducks*, taught by their classroom teacher, provoked children to visualise and to draw different solutions. Fifteen children (83%) could identify and create equal groups via drawings; eight of these children (44%) could also quantify the number of groups that were formed. These findings show that some young children can visualise multiplicative situations and can communicate their reasoning of equal group situations through drawing.

The accepted wisdom of earlier research was that the intuitive pathway for children to multiplication is through repeated addition (Anghileri, 1989). Research reported by Sullivan et al. (2001) showed a relatively large cognitive step for children to move from using models with counting to abstract multiplication. These authors recommended that the teaching of multiplication require children of 5-8 years of age to imagine objects as well as model with objects.

The theoretical framework of this research is a social constructivist theory of learning which holds that meaning is created between individuals through their interactions (Ernest, 1991). The mathematical content was framed by the research literature related to problem solving with children, early multiplication and division, and children's drawings. The ability to solve problems is a fundamental life skill and develops naturally through experiences, conversations and imagination (Cheeseman, 2018). The perceived importance of problem solving stimulates educators to look for authentic problem-solving situations in which children behave as mathematicians (Baroody, 2000). The task reported in this paper is one such non-routine mathematical problem.

Multiplicative thinking involves making two kinds of relations: the many-to-one correspondence between the three units of one and the one unit of three (Clark & Kamii 1996). Doing so requires an ability to form visual images of composite unit structures and is fundamental to multiplicative thinking (Sullivan et al., 2001). Young children are only able to abstract this notion of a composite unit when they have constructed meaning in their own minds (Bobis, 2008). In order to determine children's meaning of groups, this study used children's drawings as a research tool, and to potentially be a "window into the mind of a child" (Woleck, 2001, p. 215). Children were asked to draw a picture of what they were visualising and to describe their thinking as they solved the problem. Materials and modelling were used only when a child was unable to solve the problem (Sullivan et al., 2001). We conjectured that many children make mental images and visualise quantities when situations provoke them to do so. Our challenge was to create a context that would elicit children's thinking, and to interpret and understand what children imagine. The research question we set out to answer was: *How do children's drawings, explanations and actions reveal the ways they visualise group structures?*

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Method

A teaching experiment methodology was used to explore and explain students' mathematical actions and thoughts about recognising and making equal groups. As researchers we wanted to experience, first-hand, students' mathematical learning and reasoning (Steffe & Thompson, 2000). The study included the four basic elements of teaching experiment methodology. The "teaching episode" in this case, a sequence of five consecutive days of mathematics lessons in one school with a class of 5-6 year-olds in their first year of school. Three researchers witnessed the teaching and video-recorded each lesson.

The exploratory teaching was undertaken by Sarah (fourth author). While not privy to the team's design of learning contexts, she contributed to the theoretical framing of the study, and was conversant with the purpose of the research. Sarah was familiar with the Launch, Explore, and Summarise lesson structure (Lappan, & Phillips, 2009), and she believed that children should not be shown possible solution strategies before they attempt a task. The research team noted that the lesson content was beyond the intended curriculum and would present conceptual challenges for 5-6-year-olds, as would the exploratory teaching. Analysis of the children's mathematical thinking was based on their drawings, mathematical language and actions, and on the researchers' theoretical interpretation of events in accordance with a teaching experiment methodology. We closely observed children's interactions to infer their thinking about multiplication as seeing "groups of groups".

Participants were 21 children (13 girls and 8 boys) from a primary school in a large rural city of Victoria, Australia. The mean age was 5 years and 6 months. Sarah's class provided a convenience sample for investigating our research question. The results are from the 18 children who were present on the day. We devised lessons as contexts in which 5-6 year-old children could be stimulated to recognise and create equal groups and to quantify those groups. One lesson, Twelve Little Ducks, is the setting for the results presented here. Sarah was given a lesson outline and encouraged to implement the ideas in any way that she felt suited her children. The problem was originally written as: *Can you make 12 little ducks into equal groups? Can you do it a different way? Draw or write what you did.* To introduce the task to her children, Sarah told a story:

> In order not to lose any of her ducklings the mother duck put them into some groups that were the same. She put them into equal groups, because it was easy for her to see that she still had her 12 baby ducks. Can you make a picture in your head of those 12 little ducklings? The mother duck put them into groups with the same number of ducks in each group. I wonder what groups she put them into … I would like you to draw a picture of what is in your head (video transcript).

Sarah chose not to show a picture of ducks or to model the problem with materials, she explained that it might interfere with children's thinking. She was keen to learn what her children could imagine without objects - in a context her children would understand. Sarah was conscious of the challenge of the task's mathematical vocabulary as her diary showed:

> These children have not heard the term "equal groups" from me at school at all until today. I did say "the same number in each group" but I didn't go into great detail about what I meant by equal groups.

These pedagogical decisions deliberately created a challenging for 5-6 year-olds. Blocks were not provided initially but a child was offered blocks when it was apparent that s/he could not begin to solve the problem.

Data were collected from two fixed video cameras, three tablet cameras operated by the observer-researchers recording children working or in conversation with an adult. Subsequently, photographs of work in progress, children's finished work samples, classroom observations, and the video and photographic data were closely examined and interrogated. Data analysis began with each university researcher describing in detail what they observed soon after the lesson. In this way, we built a shared understanding of the events in the classroom. Each child's work sample was examined. Tentative categories of responses were proposed and iteratively tested to refine category definitions.

Findings

Analysis of the work samples together with our observations, conversations with the children, and video evidence revealed that three distinct categories of thinking could be described in terms of demonstrated multiplicative thinking.

Evident - *could simultaneously quantify objects in groups and enumerate the groups as new units*

Eight children (44%) produced 12 ducks by drawing and simultaneously creating equal groups. The ducks in their drawing were located in identifiable groups, indicating that they had perceived or imagined such groups before drawing the ducks. Elise drew two groups of six, circled each group and labelled her drawing, "2 groop 6" (*sic*) (Figure 1). She could make equal groups and quantify the groups. It appears Elise had determined the group size prior to drawing her solution because the ducks are drawn in equal rows.

Figure 1. Elise's first solution

Partial - *having some awareness of the quantity of each group but not the number of groups shown*

Figure 2. Georgie's first solution.

Six children (33%) were categorised as having "partial" understanding because they made equal groups but were not able to quantify the number of groups. Georgie drew three groups of four ducks (Fig 2), and when asked about her groups she said:

Georgie:	There are four here, and four there and four there. (Pointing to each group.)
Teacher:	How many groups of four have you got?
Georgie:	Twelve.
Teacher:	Twelve altogether. How many groups of four?
Georgie:	Um, I'm not sure yet.

Emergent - *unable to find a solution – even with 12 cubes to model the problem*

The four children (22%) who we described as emergent thinkers had several observed misunderstandings. For example, Conrad was unable to make six groups of two, from his drawing. It appears that Conrad did not have a solution in mind when drawing the 12 ducks as they were not drawn in identifiable clusters or rows. The random arrangement may have contributed to the difficulty of circling groups of two. Other emergent thinkers were unable to make equal groups in their drawings or when provided blocks to do so.

Figure 3. Conrad's drawing

To Conclude

We investigated whether children could visualise and construct equal groups and recognise the composite units they formed. Our research question was answered. Some children can imagine and draw equal group structures and in doing so recognize composite units. Some children can also enumerate the composite units. More than 80% of the children in the present study exhibited early multiplicative thinking. Children seemed to have intuitive understandings of equal group structures based on their experiences because they came to the problem we posed without any prior formal instruction about equal groups. This finding is novel - we have found no studies that have reported similar results with 5-6 year-old children.

Children communicated their visualisation of equal group situations through their drawings and elaborated their meaning with verbal descriptions and gestures. Such drawings of visualisations represent abstract thinking and call into question the accepted view of the way early multiplication typically develops via direct modelling to partial modelling, then to thinking abstractly (e.g., Anghileri, 1989).

We argue that it is productive to require young children to abstract problems earlier. Requiring visualisation together with drawings is an alternative approach to direct modelling. We acknowledge this is a small study and the results are only indicative of the ability of young children to visualise multiplicative situations. Further research might investigate other provocations that elicit children's thinking about multiplication. Children's drawings of their mathematical reasoning are fascinating and the intuitive understandings that young children develop about aspects of multiplication are worthy of detailed examination.

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Investigating students' drawings as a representational mode of mathematical fluency

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In sharing solutions of mathematical tasks, students may use various modes of representation such as: language (oral/written), numerical and symbolic, or drawings (pictures, diagrams or markings). In this paper we explore the potential of student drawings to provide evidence of mathematical fluency. Examples of young students' (5-8 years old) solutions to mathematical tasks are examined through the lens of drawing representations. The investigation suggested that students' drawings are valuable data when analysing work samples for evidence of mathematical fluency alongside other representations.

Drawn representations are a window into students' thinking and are worthwhile to explore in a mathematical context. Cai and Lester (2005) assert that representations not only help students make sense of mathematical problems but allow for communication of thinking to others. Bakar et al. (2016) agree that students use drawings to share solutions and suggest that "drawing was a *translation* from other types of representations, used [by students] to confirm and explain their answers" (p. 92). Within Way's (2018) research she utilised drawing to "reveal the variety in … drawings, and to explore similarities and differences across the age range" (p. 98). There exists an important transitional point during the early years of schooling for students between drawing (personal expression) and mathematical representation (function and purpose) (Bakar et al., 2016; Way, 2018). These representations require further analysis in observing students' mathematical fluency.

Data reported on in this paper is part of a larger research project (Cartwright, 2019) investigating students' characteristics of mathematical fluency and teachers' noticing of fluency. Within the study, many students produced drawings in their written work to convey their mathematical understanding in solving tasks. The drawings, as a mode of representation, became a vital aspect of analysis when observing a students' mathematical fluency. The purpose of this paper is to build on the drawing representational analysis conducted by Way (2018). In-depth analysis of the drawing work samples addresses the following research question: *How can students' drawing representations provide evidence of their mathematical fluency?*

Method

For the analysis, 39 Kindergarten to Grade 3 work samples were selected from schools involved in the research study. All students responded to the same problem: *The farmer saw 16 legs in the field. How many animals might he have seen?*

To analyse the drawings, previously researched drawing categories (Bakar et al., 2016; Way, 2018) pertaining to students' development of drawings within a mathematical context were employed. The drawing types *pictographic* and *iconic* (Bakar et al., 2016) were used to initially sort the data. Bakar et al. (2016) define drawing as pictographic "if it has realistic

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depictions of the objects stated in the problem” and iconic drawing as containing “only simple lines and shapes to embody the intended objects” (p. 89). Cartwright’s (2019) mathematical fluency characteristics were then used as an additional lens through which to view the drawings. Four fluency characteristics were used as deductive analysis categories: use of other representations (numerical or symbolic), correct process or solution, multiple solutions, and efficient strategy. Following the characteristics analysis, data were ordered into a developmental sequence based on Way’s (2018) drawing categories: picture, partial story, partition and solution.

Findings

Overall, 17 students (44%) used pictographic representations, 14 used iconic (36%), and 8 used no drawn representations (20%). Interestingly, a few students used both pictographic and iconic representations. During analysis it was necessary to split the iconic category further as a distinct difference between the way students used shapes and lines emerged. Instead of using shapes and lines to represent the animal or its legs, students used lines and circles to cordon off solutions. Some students also used lines, arrows, or circling to connect numerical solutions to symbolic or language representations (see Figure 1). The new category was named *iconic (as organisers)* to distinguish between the two uses of iconic drawings: in place of a picture, or as part of explaining the mathematical process.

The second level of analysis took the sorted work samples (pictorial features) and analysed the data using Cartwright’s (2019) mathematical fluency characteristics. All Kindergarten students (N=6) used pictographic representations. Most students also included a numerical representation. One sample included multiple solutions and the majority of students were able to use an efficient strategy to count the legs (see Figure 2). Most students obtained the correct number of legs (16) but did not mention the number of animals.

Figure 1. Example of using lines and circling to organise solution

Figure 2. Kindergarten example of counting by ones

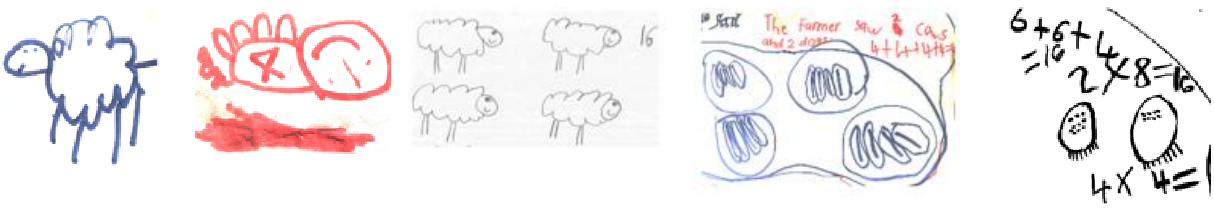
The Grade 1 samples have not been reported on in this paper as there were only three work samples, not enough to make significant statements. For Grade 2 (N=22) twenty of the students included a numerical representation to support their process or solution. Students used pictographic and iconic drawing types, however, there were significant differences in the mathematical features across the samples. One significant difference was the use of symbolic representation. Almost all students who used no drawings included symbols. Whereas only a few students who drew pictographic or iconic representations used symbols. Another significant difference was with solutions and types of efficient strategies. Most students who used pictographic representations did not produce multiple solutions and showed no strategy or an additive strategy. Compared with students who drew iconic representations or no drawings where multiple solutions and higher strategies (multiplicative) were observed. All but one Grade 3 sample (N=8) used numerical representations and six included symbolic representations as well. Most students recorded a correct process and solution and the majority of students used multiplicative strategies. Of the students who drew pictographic representations, none produced multiple solutions. Students who drew iconic representations or used no drawings were able to produce multiple solutions, often using their knowledge of number patterns to find different combinations.

Way's (2018) developmental sequence was used in analysing both pictorial and mathematical features of the work samples. Levels (described in Table 2 and illustrated in Figure 3) were adapted as the analysis progressed.

Table 2.
Developmental Sequence of Mathematical Drawings (Adapted from Way, 2018)

Level	No.	Level description
1. Scribble	0	Incoherent, no representation of the mathematical story
2. Picture	2	Shows pictures from the story problem (i.e. animal, farm) but no numerical labels or symbolic representations attached
3. Emergent Story - incorrect process/ solution	2	Shows pictures or iconic representations of the story and includes numerical values. No correct mathematical process or solution are visible.
4. Partial Story - errors with process or solution	7	Uses pictures or iconic representations and numerical values to show process of solving the problem. Correct process but incorrect/incomplete solution. Or correct solution with incomplete/incorrect process.
5. Partition and Solution	7	Uses pictures or iconic representations and numerical values during the process. Shows a correct solution.
6. Advanced Partition and Solution	13	Uses pictures or iconic representations and numerical values during the process. May include multiple solutions or patterns to find solutions.

N=31 (students who did not use drawings have not been included within this analysis)



Level 2 Level 3 Level 4 Level 5 Level 6

Figure 3. Illustrations of mathematical drawing levels

The use of a developmental sequence was beneficial when analysing the mathematical fluency features. For example, both Ellen and Daniel (Figures 4 and 5) used pictographic representations and in the initial analysis were grouped together. However, once these student samples were analysed using the developmental sequence of drawing levels, differences in their use of the representations appeared. Ellen used pictographic and iconic representations in an advanced way compared to Daniel. She labelled her pictures numerically which aligned to her cumulative count by fours. Ellen also drew lines to explain her partitioning of 16 into eighths, then fours to describe her process. Although Daniel used a correct process and found a correct solution, his pictographic and numerical representations were separate. It is unclear if Daniel made a connection between the animals' legs and his count of four. Both samples show characteristics at *Level 5: Partition and solution.* However, if we see the drawings along a continuum of development, Ellen's would be placed higher.

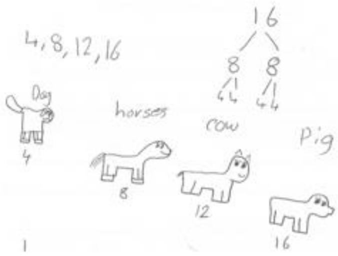


Figure 4. Ellen's work sample

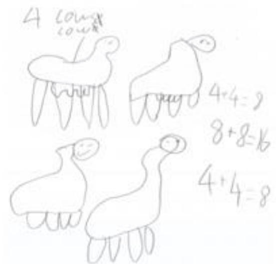


Figure 5. Daniel's work sample

Discussion and Conclusion

It was clear that drawing ability by itself did not always correspond to a student's mathematical understanding. However, students who made direct links between drawings, numerical, and symbolic representations, showed a higher level of mathematical fluency. The findings suggest that there are both affordances and issues with utilising students' drawings to analyse their mathematical fluency. One benefit was that drawings were a visual depiction of students' mathematical strategies. The way students grouped animal legs or drew arrays assisted in deciding if students were applying additive or multiplicative thinking, especially when the symbolic representations were not present. Some impacting factors emerged. Drawing ability *was* an issue for students unable to draw animals appropriately, i.e. incorrect number of legs. For students who drew pictographic representations time was a factor. The time it took to draw the animals resulted in only one solution being found, whereas students who used iconic representations generally found multiple solutions. Future research could explore iconic drawing further, specifically when students created array structures, and could be aligned to Mulligan and Mitchelmore's (2013) levels of Awareness of Mathematical Pattern and Structure (AMPS). Iconic representations revealed students' knowledge of number structure and provided scaffolding to efficiently solve the task.

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Children's drawings as a source of data to examine attitudes towards mathematics: Methodological affordances and issues

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Ascertaining young children's attitudes towards mathematics has its challenges. Methodologically, limitations exist regarding the type of research techniques that can be employed. The use of children's drawings as a data source has both methodological affordances and issues. The study was conducted with 106 children in Years 2 and 3 from three South Australian primary schools. This paper identifies some of the methodological affordances and issues of using children's drawings to ascertain and describe their attitudes towards mathematics.

For Vygotsky, a "young child's creative forces are concentrated on drawing not by chance, but because it is precisely drawing that provides the child with the opportunity to most easily express what concerns him at this stage" (Vygotsky, 2004, p. 43). Children's drawings act as a list or "graphical narration" about what a child is portraying (Vygotsky, 2004, p. 77). Numerous researchers have used children's drawings in the mathematics domain. However, few researchers have used children's drawings to ascertain and describe young children's attitudes towards mathematics. Bobis and Way (2018) state that "representations are an integral part of learning mathematics" (p. 56) and while these authors refer to representations primarily from a conceptual and working mathematically perspective, children representations of themselves are ubiquitous in their drawings. This research connects the ubiquitous nature of children's drawings of themselves with mathematics education by asking children to draw themselves "doing mathematics" as a means of ascertaining their attitudes towards mathematics.

The use of children's drawings is an innovative approach to ascertain an individual's attitude which moves away from traditional research methods such as attitudinal questionnaires. The use of children's drawings provides several affordances that traditional research methods do not allow, including providing a method to children to voice their attitudes which can then describe the nature of their attitudes in depth. Conversely, the innovative nature of this research raises several issues related to the interpretation and analyses of children's drawings. This paper examines some of the affordances and issues of using children's drawings to ascertain young children's attitudes towards mathematics.

The purpose of this study was to investigate the attitudes of young Australian children in Years 2 and 3 have towards mathematics. This investigation answered the broad question: *What are the range and nature of attitudes young children exhibit towards mathematics, in both lesson and non-lesson contexts?* It is essential to distinguish between the range and nature of young children's attitudes towards mathematics. In this paper, a distinction has been made to ensure clarity around the two words. Additionally, the words 'nature' and 'range' have often used interchangeably, but both describe specific aspects of this research. The range refers to the scope or extent of young children's attitudes towards mathematics, providing a broad view of the issue. The nature of attitude is descriptive, providing the basic

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qualities, structure, and the essence of individual attributes of children's attitudes towards mathematics. In other words, the nuances or fine-grain view of attitudes.

Method

This paper discusses findings from the non-lesson context where children drew a picture of themselves doing mathematics, provided a written description of their drawing and participated in a semi-structured interview. One hundred and six children, aged between 7 and 9 years of age, participated in a mixed-method research design where children's drawings started a conversation about their attitudes towards mathematics.

Utilising the work of Bachman et al. (2016) the prompt "Draw yourself doing mathematics" was given to participants on an A3 piece of paper. The researcher read a prompt (see Quane et al., 2019) to children with no time limit given to children to produce their drawing. Children provided a written description of their drawing and then participated in a semi-structured interview. Using the three research techniques is viewed as "complementary methods" to "understand children's lived experiences" (Macdonald, 2009, p. 48). The generated data from the three research techniques was analysed using a modified version Three Dimensional Model of Attitude (TMA) (Zan & Di Martino, 2007). The original TMA framework comprised of three aspects of attitude: an emotional dimension; a vision of mathematics; and perceived competence. In the discussion below we take up the methodological affordances of using children's drawings in terms of TMA, in the course of our research.

Findings and Discussion

The use of children's drawings was effective in identifying the range and describing the nature of young children's attitudes towards mathematics. However, while the use of children's drawing as a research technique has benefits, it raises some issues. In this discussion, the affordances and issues pertaining to the use of children's drawings is reviewed.

Attitude is a multi-dimensional construct (Zan & Di Martino, 2007) that can be complex to unpack. Any research method employed to ascertain attitudes towards mathematics needs to disentangle the different strands of this complexity. That is, the use of children's drawings as a research tool needs to be sensitive to the multi-faceted nature of the construct in question, namely attitude. Additionally, data about attitudes towards mathematics has to capture the dynamic interplay between the dimensions of attitudes.

Drawings constitute an accessible vehicle for communication, expressing what is important for the child. Unlike surveys, drawings are open-ended, expressive and are child-centred tasks (Stiles et al., 2008). Stiles and colleagues (2008), found that "attitudes towards mathematics expressed in drawings significantly correlated with attitudes expressed in the TIMSS [The International Mathematics and Science Study] statements about mathematics" (p. 1) and are "superior to the TIMSS statements" (p.13).

Drawing affords children to express what is important to them in a medium that they feel comfortable. Further, children could express a variety of emotions, as shown in Figures 1 – 3. Children articulated connections between the emotions that they expressed to specific mathematical topics and their perceived competence in mathematics.

The second dimension of attitude is children's vision of mathematics (Di Martino & Zan, 2011). For this research, children's vision of mathematics was characterised by the topics, tasks, and processes that they depicted and described as well as their value and appreciation.

The use of children's drawings provided insights into children's vision of mathematics in terms of how children depicted the mathematics that they were doing. The drawings show the interconnectedness of the three dimensions of attitude with children indicating their emotion and self-concept. Figures 4 – 6 show the mathematical topics and the children's representations of these topics. Further data from the non-lesson context provided insight into children's perceived competence, particularly their mathematical mindset and self-concept. For example, C16 (Figure 1) indicated that she hated mathematics, finds it hard but wants to try "make friends" indicating she has a low perceived competence in mathematics.

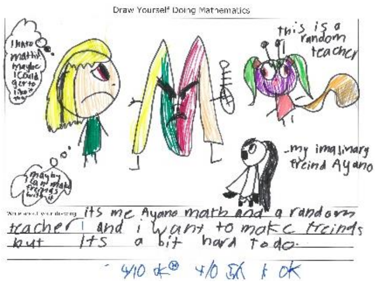

Figure 1. C16; female, negative attitude

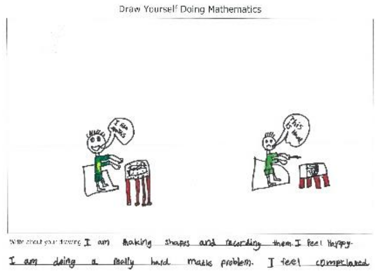

Figure 2. B8; male, neutral attitude

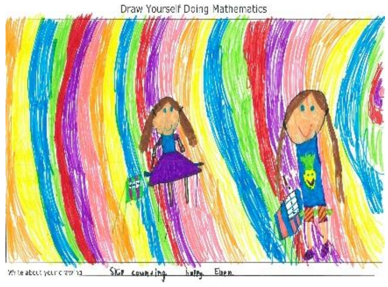

Figure 3. A25; female, positive attitude

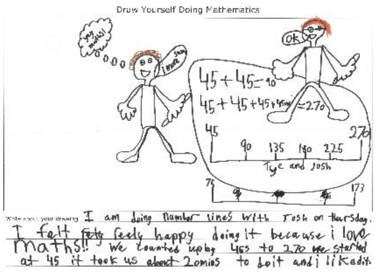

Figure 4. A13; male, extremely positive attitude

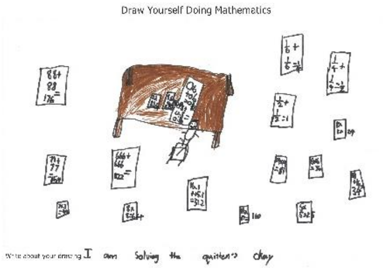

Figure 5. B45; male, positive attitude

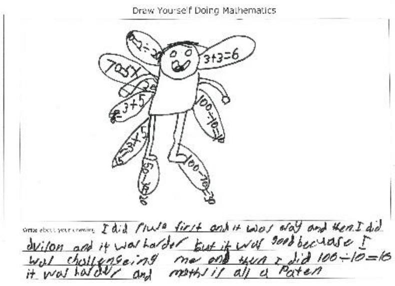

Figure 6. C6; male, positive attitude

Lowenfeld and Brittam (1964) were instrumental in describing the developmental nature of children's drawings. In so doing, these authors drew attention to the principle of 'deviation' as a means for children to emphasise, exaggerate or omit pictorial elements. It is important to note how an observer views these three principles. Lowenfeld and Brittam (1964) cautioned the observer of a drawing regarding making incorrect judgments about a child's intention of using disproportional elements within a drawing. Correct judgements and interpretations can only be made by asking the child about their drawing to understand the reasons for using disproportionally or drew a particular object. When children have used the three types of deviations, the child has drawn what is real, significant, and relevant to them (Lowenfeld & Brittam, 1964).

The principle of deviation is seen in A25's drawing (Figure 3), where she has emphasised the background of her drawing. The child explained that she loved patterns. The emphasis that the child placed on her rainbow background would not have been realised without asking the child open-ended questions about her drawing. The background in A25's drawing consumed A25's attention and focus including her responses to the interview questions. Understanding the importance A25 has placed on the background was required to minimise the potential for the generation data that may have been unreliable. Asking the child about the other elements within her drawing and other open-ended questions such as "what is maths?" provided indicators for all three dimensions of her attitude.

A second emerging issue with using children's drawings as a research technique is the interpretation. The following example illustrates the potential for misinterpretation. Two boys have used the same colour for their face, but the reasons for their colour choice is very different. B17 (Figure 7) has chosen the colour as he believes it reflects his skin colour. B42 (Figure 8) has chosen the colour to show that he is feeling frustrated. Examining the drawings in isolation from the other data sources may produce very different conclusions. It is only when the child is asked about what they have drawn and why they have chosen to draw it in the way that they have, do we truly understand the meaning in their drawings.

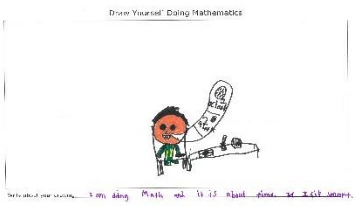

Figure 7: B17; male, extremely positive attitude

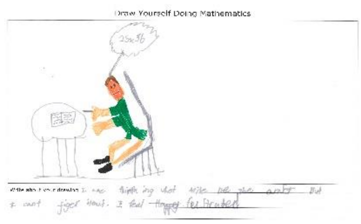

Figure 8: B42; male, neutral attitude

Conclusion

The use of 'Draw yourself doing mathematics' elicits children's drawings that were personal stories about their complex relationship with mathematics revealing their attitude towards mathematics. The process of drawing was a means for children to feel comfortable sharing their thoughts in a familiar manner (Macdonald, 2013). Children were given the time to "comprehensively explain the intended meanings of their drawings through extended conversations and further questioning" (Macdonald, 2013, p. 72). An affordance not offered in quantitative measures. Children's written responses complemented the visual and verbal accounts adding further insights into what was important to them. By providing children multiple opportunities to share their thoughts about mathematics, rich narratives were told about individual attitudes towards mathematics.

In conclusion, our experiences thus far showed that there are challenges in using drawings particularly in unpacking the developmental aspects of attitude. On balance, however, the affordances outweigh the hindrances in deploying the technique. The affordances of using children's drawings can be summarised as giving children the freedom to choose what they depict and how they portray themselves. For children's drawings to be understood by adults, Anning and Ring (2004) offer the following: "We need a society that can listen to children and recognise that perhaps their drawings may tell us much more about childhood than we ever imagined" (p 125).

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